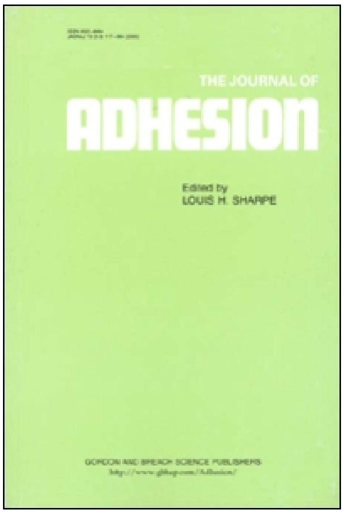


The Journal of Adhesion
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453635

Local Bending Moment as a Measure of Adhesion: The Blister Test

Olga A. Goussev[a]; Karel Zeman[a]; Ulrich W. Suter[a]
[a] Institut für Polymere, Eidgenössische Technische Hochschule, Zürich, Switzerland

To cite this Article Goussev, Olga A. , Zeman, Karel and Suter, Ulrich W.(1996) 'Local Bending Moment as a Measure of Adhesion: The Blister Test', The Journal of Adhesion, 56: 1, 45 — 57
To link to this Article: DOI: 10.1080/00218469608010498
URL: http://dx.doi.org/10.1080/00218469608010498

J. Adhesion, 1996, Vol. 56, pp. 45–57
Reprints available directly from the publisher
Photocopying permitted by license only

Local Bending Moment as a Measure of Adhesion: The Blister Test

OLGA A. GOUSSEV, KAREL ZEMAN, and ULRICH W. SUTER

Institut für Polymere, Eidgenössische Technische Hochschule, CH-8092 Zürich, Switzerland

(Received May 24, 1995; in final form August 29, 1995)

We propose to characterize joints between materials by the maximum bending moment, M_{max}, borne just prior to delamination. We suggest to evaluate M_{max} in the blister test geometry through direct measurement of the blister curvature in the vicinity of the separation line and employ a scanning capacitance microscope for the blister profiling. The methodology and apparatus were tested on measurements of adhesion of two commercial polymer films to Plexiglas and Teflon.

KEY WORDS: Adhesion; adhesion energy; adhesive fracture test; blister test; elastic plate behavior; critical reaction moment; local bending moment

INTRODUCTION

The term "adhesion", in its colloquial use, describes the tendency of dissimilar materials to "stick together" when they are in intimate interfacial contact. A quantitative definition is more difficult to formulate since, in practice, adhesion comprises geometrical and rheological contributions in addition to (and often far exceeding) the "intrinsic" part which is, nevertheless, necessary for surfaces to "stick" together.[1]

To analyze the process of fracture in a homogeneous material quantitatively, Griffith[2] suggested an approach based on continuum fracture mechanics in which the problem was addressed by assuming that the fracture strength of brittle, elastic materials is due to the growth of preexisting heterogeneities ("flaws") into cracks. An alternative approach to failure prediction, based on using the volume energy density as a fracture criterion instead of the surface energy density used by Griffith, has been proposed by Sih.[3] The similarity of adhesive and cohesive fracture, and particularly the energy concept of fracture, has been elucidated from a continuum mechanics viewpoint by Williams.[4] Most later work relies on this basis as does much of the discussion below.

In almost all real systems the fracture process is irreversible to an appreciable degree and is accompanied by dissipative processes so that the real adhesion is determined largely by the nature of existing flaws and visco-elastic properties of the system. The fracture energy can be considered to consist mainly of reversible work of adhesion (W_r) and irreversible plastic work (W_p):

$$W = W_r + W_p \tag{1}$$

For a perfectly brittle material, where plastic yielding does not occur, $W = W_r$. This is a good approximation for silicate glasses and oligomeric polystyrene far below T_g.[5] However, for most materials, even for brittle ones, the plastic work is usually comparable with, or greater than, the reversible work of adhesion, sometimes by several orders of magnitude,[5] so that the measured value of W is dominated by W_p.

We do not attempt here to separate the reversible from the reconstitutive, irreversible, dissipative part of practical adhesion but, rather, as an alternative to the energy approach in the practical characterization of the adhesion between two solids, propose to characterize the joints between materials directly by the maximum bending moment, M_{max}, borne just prior to delamination.

BLISTER TEST GENERALITIES

These exists a great variety of mechanical tests to deal with practical adhesion;[1,5–7] examples are butt joint tests, shear tests of lap joints, peel tests, cantilever beam tests, cone tests, or blister tests.

We shall be dealing in this work only with the blister test (see Fig. 1) that is well suited for measurements of adhesion of polymer films to solid substrates.[7] In this test, a fluid, gas or liquid, is injected under the film thereby forming a blister, and the hydrostatic pressure is increased until the film begins to detach from the interface.[8] In practice, if the film is too thin or adheres too strongly, the blister may burst before peeling is initiated. To overcome this problem, the "island"[9] and the "peninsula"[10] blister geometries have been proposed. The "inverted-blister" test method[11] is another modification well-suited for thin polymeric films.

Quantitative analysis of blister-test measurements can be based on Griffith's hypothesis. The peel angle (*i.e.*, the angle between film and substrate at the locus of delamination) is small and it is, therefore, usually assumed that the plastic and visco-elastic components of the adhesion energy can be neglected to a first approximation, and that the change in stored elastic energy as the blister grows can simply be equated to the energy required to separate the adhering film from the substrate. This is usually formalized by assuming that, away from the crack tip region, the film may deform in one of three principal mechanisms: (i) for a relatively thin film ($t \ll 2a$, where t is the thickness and a is the radius of the blister), deformation is considered to be mainly tensile and the blister is modelled as an elastic membrane with negligible bending resistance—the major source of stored elastic energy arises from the stretching of the film, and the blister height at its center is proportional to the 1/3 power of pressure; (ii) for a relatively thick film ($t \approx 2a$), the blister is considered to be deformed mainly by bending and is modelled as an elastic plate with a rigid edge constraint ("clamped plate")—the major source of stored elastic energy is the bending deformation (here deflections are small compared with the thickness of the film and the blister height at its center is proportional to the pressure); (iii) for a thick film ($t \gg 2a$) the film is modelled as an infinite medium where the elastic energy is predominantly stored in the highly-stressed regions around the edge of the blister. Another contribution to the stored elastic energy that influences the process of delamination arises from residual stresses inherent in the laminate; their contribution to the delamination mechanics has been considered by Senturia *et al.*[9,12] and found to be of significance.

The above mentioned limiting conditions allow analysis through the theory of elasticity assuming that the adhering film is homogeneous, behaves elastically over the entire range of deformation, and that Poisson's ratio is known and remains constant during the entire experiment. A severe restriction is the number of experimental quantities that can be obtained in a blister test. In all published methods, the pressure of the injected fluid is monitored and its value immediately before delamination of the blister, P_{cr}, is taken as the characteristic limit. The various published methods are then distinguished by the limiting case taken in the elastic theory and by the additional experimental values required: the volume of fluid injected until delamination, ΔV, the height of the blister at the instant before delamination, h_{cr}, the radius of the blister just before delamination, a (here, we limit ourselves to circular blisters, although several other forms have been described), and the thickness of the film, t. Many solutions require, in addition, material constants such as, commonly, Young's modulus, E, and Poisson's ratio, ν. We list a number of published approaches in Table I. All of them are based on an energy-balance approach and neglect dissipative processes. Their respective merits and limitations have been discussed in several publications.[10,11,17] (The

TABLE I

Some formulae available from the literature for calculation of the adhesion energy from circular blister test data. All models assume elastic materials and quasi-static experiments

Source	Ref.	Adhesion Energy[a]	Model & Assumptions
Dannenberg, 1961	8	$W=\int_0^{P_{cr}} P(V)\,dV \approx P_{cr}\Delta V$	Estimation of total work to delamination
Williams, 1969	4	$W=\frac{3}{32}\frac{(1-\nu^2)P_{cr}^2 a}{E}\left(\frac{a}{t}\right)^3$	Bending of a thick plate, radius used
		$W=0.5\,P_{cr}h_{cr}$	Same, height used
Andrews & Stevenson, 1978	13	$W=\frac{(1-\nu^2)P_{cr}^2 a}{E}\left[\frac{3}{32}\left[\left(\frac{a}{t}\right)^3+\left(\frac{a}{t}\right)^2\frac{4}{1-\nu}\right]+\frac{1}{\pi}\right]$	Bending of a thick plate
Hinkley, 1983	14	$W=0.25\,P_{cr}h_{cr}$	Stretching of a thin membrane
Takashi & Yamazaki, 1978	15	$W=\left(\frac{P_{cr}^4 a^4}{18.2Et}\right)^{1/3}$	Stretching of a thin membrane
Gent & Lewandowski, 1987	16	$W=\left(\frac{P_{cr}^4 a^4}{17.4Et}\right)^{1/3}$	Stretching of a thin membrane, radius used
		$W=0.65\,P_{cr}h_{cr}$	Same, height used
Briscoe & Panesar, 1991	17	$W=\left(\frac{P_{cr}^4 a^4}{576(1-\nu)^2 Et}\right)^{1/3}$	Stretching of a thin membrane

[a] P_{cr} pressure of the injected fluid immediately before delamination of the blister
h_{cr} height at the center of the blister immediately before delamination
a radius of the blister
t thickness of the film, E Young's modulus, ν Poisson's ratio

large discrepancy in adhesion energy between formulae from Ref. 15 and 16 *versus* 17 arises because of the way the strain energy is calculated[17]). In the following, an alternative will be proposed.

AN ALTERNATIVE METHOD OF ANALYZING THE BLISTER TEST

We follow Obreimoff's elegant approach[18] which he utilized to estimate the cohesive strength of mica. To this end we consider the blister in Figure 1. The equations of mechanical equilibrium for the polymer film can be written as follows:

$$\frac{\partial \sigma_{ik}}{\partial x_k} + \rho g_i = 0 \qquad \sigma_{ik} n_k = F_i \tag{2}$$

where σ_{ik} is the stress tensor ($\{i,k\}^2 \in \{1,2,3\}^2$), n_k is a unit vector normal to the surface, F_i is the force distributed over the surface of the film, ρ is the film's density, and g_i is the gravitational acceleration vector. The arrangement is subject to "clamped boundary conditons":

$$z\Big|_{\text{boundary}} = 0 \qquad \frac{\partial z}{\partial n_i}\Big|_{\text{boundary}} = 0 \tag{3}$$

where z is the deflection of the film from the plane of the substrate.

If the stress-strain relationships are known, one can use Finite-Element methods to compute the maximum bending moment at the separation line of the blister, M_{max}. This bending moment is of special significance since it directly describes the mechanics working against adhesion at the vulnerable edge of the blister: where the separation line between the materials is at rest, the bending moment, M, at the separation line must be equal to the reaction moment which is itself a *direct* physical characteristic of the adhesion between materials. Accordingly, we propose to add to our tool chest the local

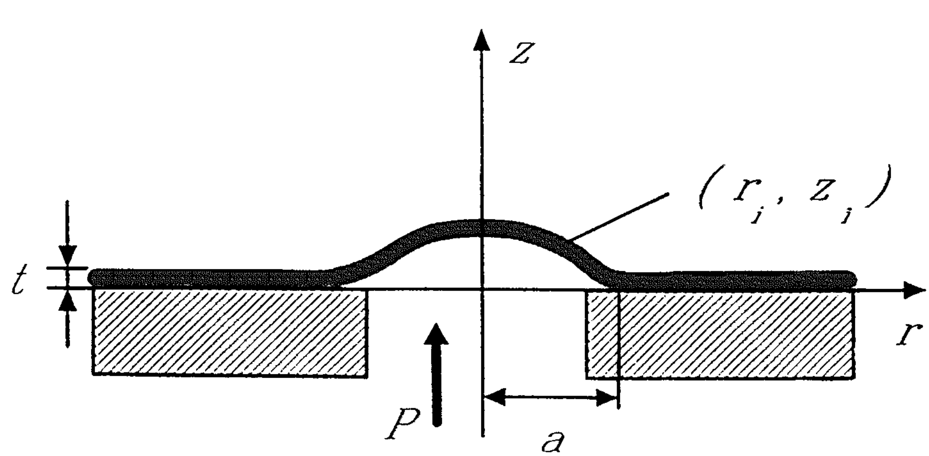


FIGURE 1 Schematic of the blister test.

moment balance condition at the separation line at the instant before delamination, thus avoiding the use of the energy balance approach where the contributing parts are intrinsically non-local and some of them are always unknown.

We equate the reaction moment at the separation line to the blister's bending moment measured in the internal proximity of the separation line of the blister. Close to the separation line, the displacements in the z direction are small. Hence, plate theory should be applicable and, for clamped boundary conditions, the bending moment is determined by the second derivative of the deflection, z, along the normal to the contour[19]

$$M \propto \left.\frac{\partial^2 z}{\partial n^2}\right|_{\text{boundary}} . \tag{4}$$

The bending moment can be determined from the measurement of the deflection, z, near the separation line, the proportionality constant being dependent on the geometry considered.

For the case of circular blister geometry (see Fig. 1), the value of the second derivative at the separation line can be approximated by series expansion of the blister's shape, $z(r)$, in the vicinity of the separation line, $r \leq a$:

$$z(r) = z|_{r=a} + \left.\frac{\partial z}{\partial r}\right|_{r=a}(a-r) + \frac{1}{2}\left.\frac{\partial^2 z}{\partial r^2}\right|_{r=a}(a-r)^2 + O((a-r)^3) \quad r \leq a \tag{5}$$

where a is the radius of the blister. Because of clamped boundary conditions [see Eq. (3)] the first two terms are zero so that

$$z \approx \frac{1}{2}\left.\frac{\partial^2 z}{\partial r^2}\right|_{r=a}(a-r)^2 \quad r \leq a. \tag{6}$$

If one can measure the blister shape and, hence, the film's deflection as a set of value pairs, $\{r_i, z_i\}$, in the vicinity of the separation line, one can evaluate the second derivative at the separation line by fitting Eq. (6) to the measured deflections, z_i, at radii, r_i, near the separation line using minimization of the mean-square deviation

$$\Phi = \sum_{\substack{i \\ z<t, r_i<a}} \left[\frac{1}{2}\left.\frac{\partial^2 z}{\partial^2 r}\right|_{r=a}(a-r_i)^2 - z_i\right]^2 \tag{7}$$

(the condition $z < t$ constitutes a safe limit).

From the second derivative at the separation line one obtains the bending moment, M

$$M = D\left(\left.\frac{\partial^2 z}{\partial r^2}\right|_{r=a}\right) \tag{8}$$

where D is the flexural rigidity of the polymer film. This relationship, Eq. (8), is valid for any elastic material, *e.g.*, also for a multilayer laminate. If, in addition, the film material is homogeneous, the flexural rigidity, D, can be written in terms of common material properties:[20,19]

$$D = \frac{Et^3}{12(1-\nu^2)} \tag{9}$$

where E is Young's modulus, ν is Poisson's ratio and t is the thickness of the film. The bending moment just before delamination, M_{max}, is taken as the characteristic of the delamination process. Quantitative interpretation of the measurement of the critical curvature of the blister at the separation line requires knowledge of the flexural rigidity, D. For the purpose of comparing systems with identical film material and different adhesion, however, it is sufficient that D is the same in all systems considered.

Having at hand the value for the critical curvature, one can estimate the adhesion energy using the energy balance approach:[18,19] the work done by the bending moment when the radius of the separated surface increases by δr is set equal to the change in the energy of the system.[18,19] The latter is made up of two parts: the change in the surface energy and the change in the elastic energy. Assuming dissipation to play an irrelevant role this leads to the following formula for the adhesion energy,[19] W

$$W = \frac{1}{2} D \left(\left. \frac{\partial^2 z}{\partial r^2} \right|_{r=a} \right)^2_{max}. \tag{10}$$

Obreimoff[18] proposed the same formulae for the bending moment and adhesion energy as in (Eqs. (8) and (10)) but with a different flexural rigidity; including Poisson's ratio, it is:

$$D_{OB} = \frac{Et^3}{3(1-\nu^2)}. \tag{11}$$

The difference between the two approaches (Eqs. (9) and (11)) arises from the assumption about where the neutral surface is situated. In the Landau-Lifshitz approach[19] it is assumed to lie midway through the plate, whereas in Obreimoff's analysis[18] it is supposed to be at the lower surface of the torn-away film. We believe that Obreimoff's analysis is more suitable for the calculation of W in our case so that the appropriate equation for adhesion energy is:[18]

$$W = \frac{1}{2} D_{OB} \left(\left. \frac{\partial^2 z}{\partial r^2} \right|_{r=a} \right)^2_{max} = \frac{1}{6} \frac{Et^3}{(1-\nu^2)} \left(\left. \frac{\partial^2 z}{\partial r^2} \right|_{r=a} \right)^2_{max}. \tag{12}$$

EXPERIMENTAL

A scanning capacitance microscope[21] has been used for the measurement of the blister profile. This instrument provides a profile of a surface without contact: A fine wire probe is scanned over the surface of the sample under examination (Fig. 2) while the electrical capacitance of the system consisting of the needle and the sample is maintained at a constant value, thereby maintaining a fixed distance between needle tip and surface. The relative change in the height of the probe during scanning faithfully replicates the sample profile. We employed a modified SCP7000 noncontact surface

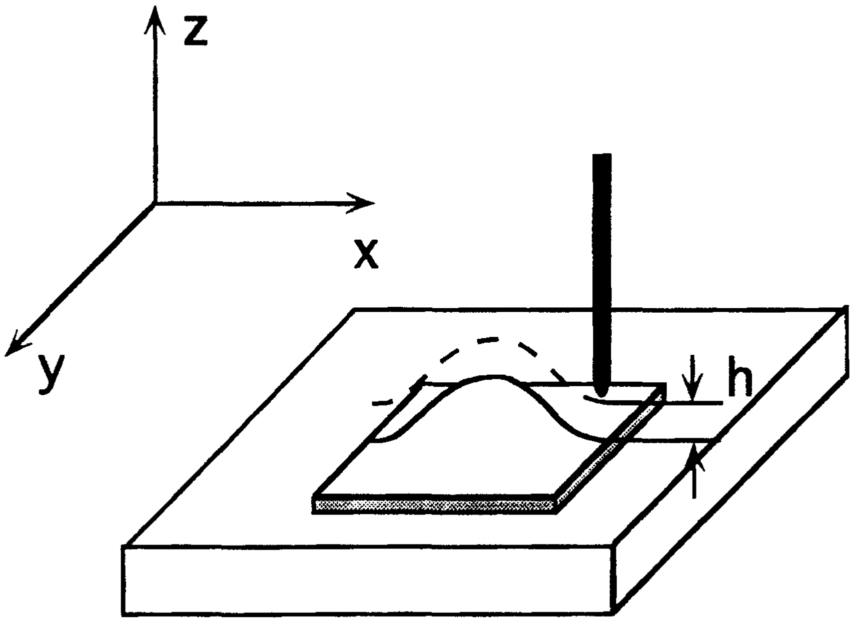


FIGURE 2 Schematic of the scanning capacitance microscope employed.

profiler of Wentworth Laboratories Ltd with a vertical resolution of approx. 0.1 μm and a horizontal resolution of 10 μm.

For blister test measurements, the sample, consisting of the film on a substrate with a hole, is fixed to the instrument's table by vacuum. The sample is placed so that the scanning line crosses the center of the hole.[22] A given pressure of nitrogen is then applied to raise a blister and the blister shape is measured. The pressure is then increased and the measurement performed again. This process is repeated until failure occurs.

With the measured values of blister height, h_{cr}, and radius, a, just before delamination at pressure P_{cr}, one can estimate the value of the adhesion energy using equations currently available in the literature (see Table I). From the blister profile at the peeling pressure one can also evaluate the second derivative at the separation line and calculate the maximum bending moment, M_{max}, and the adhesion energy, W, as given by Eq. (12).

As test samples, two commercial adhesive tapes (3M Co. Tape #35, $E = 5.0$ MPa and Tape #375, $E = 1.2$ GPa)[16] were employed. The tapes were applied to the surface of commercial slabs of Teflon and Plexiglas which had been previously cleaned with methanol and dried. The tapes were pressed onto the substrates by hand.

To permit the needle of the scanning capacitance microscope to sense the surface more accurately, a thin copper layer (20–60 nm) was evaporated onto the film. Previous experiments with copper layers of thickness up to 300 nm indicated that metal layers of this thickness do not influence the measured values.

RESULTS AND DISCUSSION

A typical experimental profile is plotted in Figure 3. The points with deflections less than the thickness of the film (indicated by a horizontal dotted line in the figure) were taken for the calculation of the second derivative *via* Eq. (7) using a horizontal baseline for points outside the blister ($r > a$); the three fitting parameters were the second derivative, the locus a, and the height of the baseline. As can be seen from the picture, the quadratic approximation (Eq. 6) is well suited for these films.

Figure 4 displays, as a function of applied pressure, the blister diameter and the average of the second derivatives of the two sides of the blister. The error bars indicate the difference between the curvatures measured at the two sides of the blister and characterize the blister asymmetry. It has to be noted that the blister symmetry is sufficient for the intended accuracy, the difference between the two sides of one blister being typically less than 30% and often less than 5%. One also sees that, especially for Tape #375, delamination does not occur precisely as one would expect. Rather, it seems to be common that the blister starts to broaden with increasing pressure, similar to observations published in earlier work.[16] Once the delamination starts, the value of the

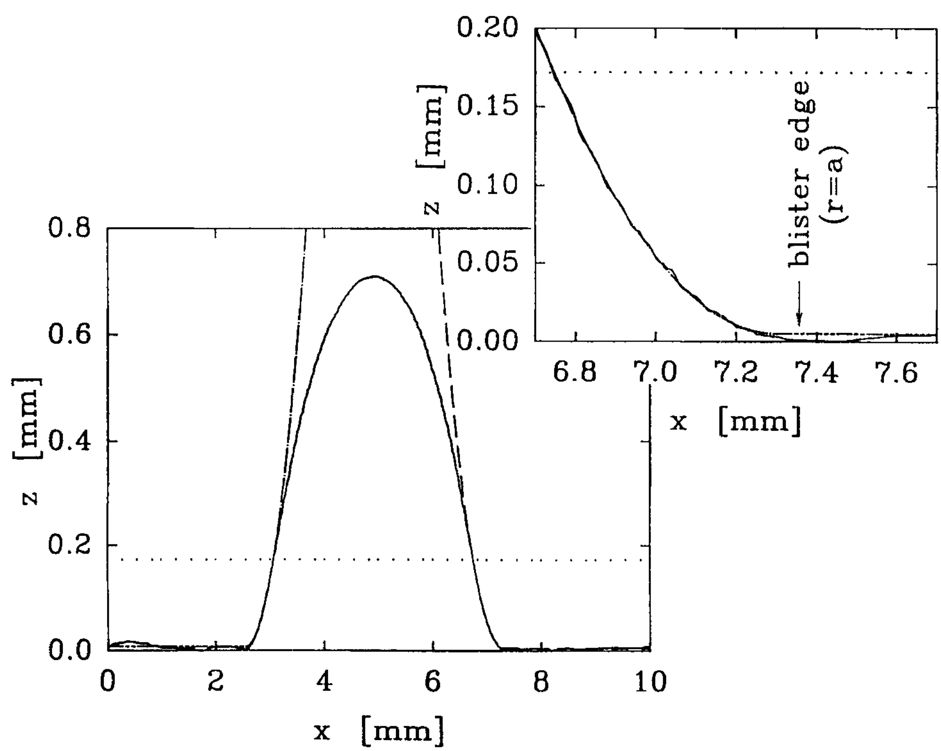
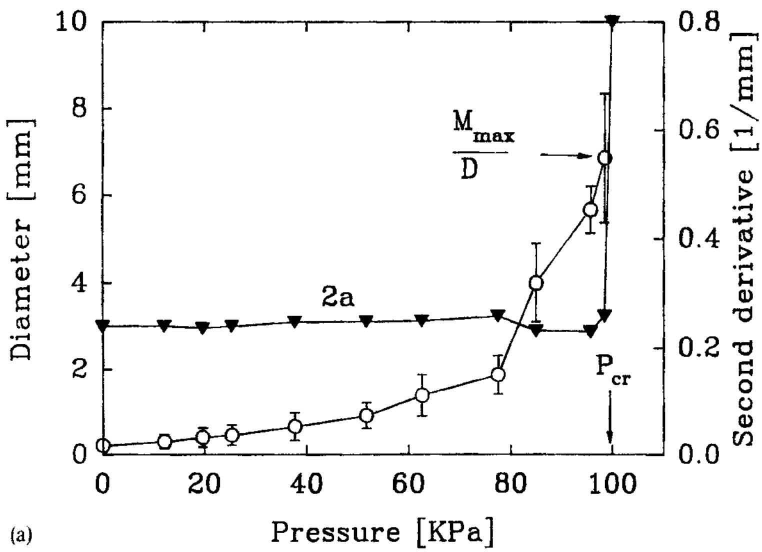


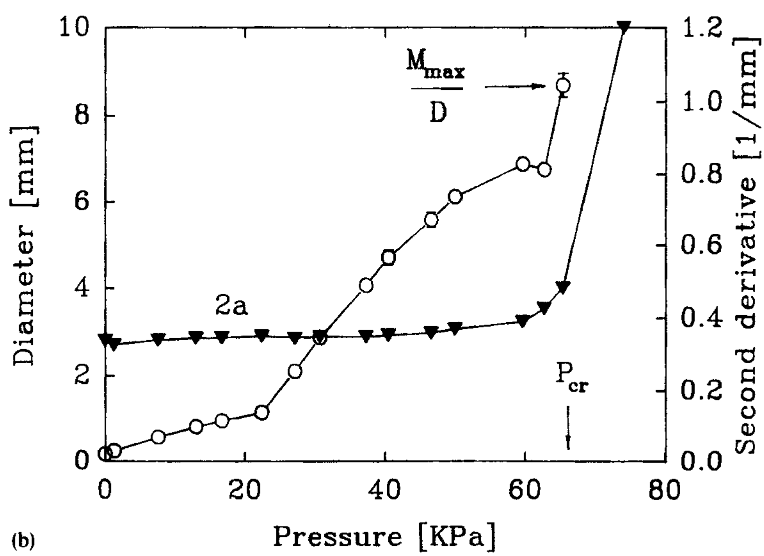
FIGURE 3 Solid line: Experimental profile for Tape #35 on Teflon. Dotted horizontal line: Thickness of the film (0.172 mm). Dashed and dash-dotted lines: Approximation parabolas and horizontal baselines for the left and right sides of the experimental profile.

measured bending moment seems to remain approximately constant during the delamination (Fig. 4d) indicating that this quantity is a physical characteristic of the delamination process.

Oue experiments were performed without control of the debonding rate and, hence, without control of the viscoelastic processes occurring during debonding. In fact, it is

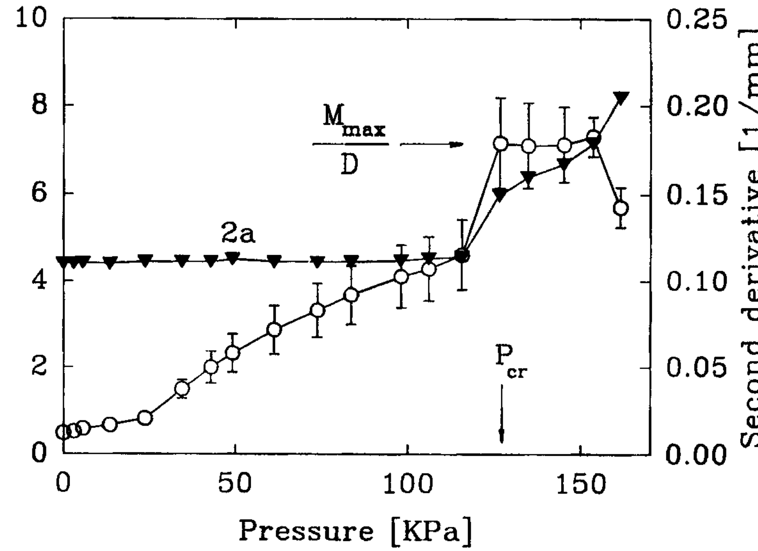


(a)

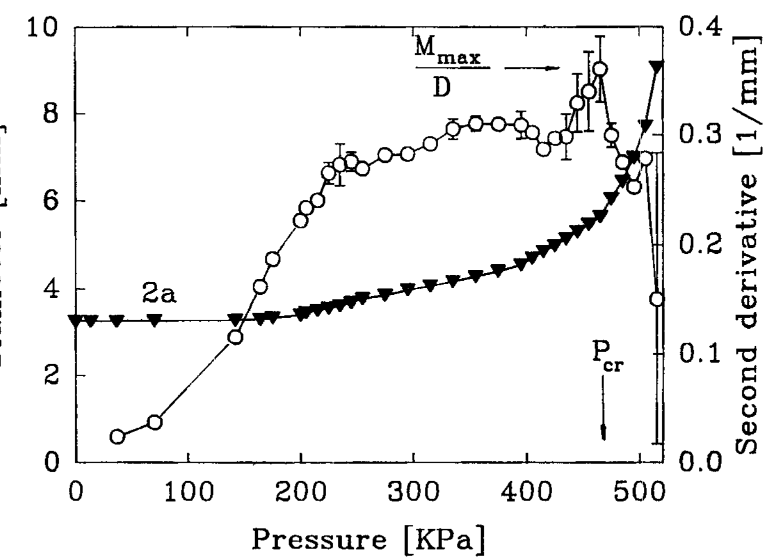


(b)

FIGURE 4 Diameter $2a$ (triangles) and the average second derivative $\partial^2 z/\partial r^2$ at ($r = a$) of the experimental profiles (circles) *vs* pressure P: a) Tape #35 on Teflon, b) Tape #35 on Plexiglas, c) Tape #375 on Teflon, d) Tape #375 on Plexiglas. The error bars correspond to the difference between the curvatures measured at the two sides of the blister.



(c)



(d)

FIGURE 4 (*Continued*).

not clear how such a control could be exercised, since the response of the blister to a step increase in pressure is fast.

From the data presented in Figure 4, it is possible to define a critical pressure as that pressure where the second derivative has its maximum. From the pressure, height, and radius at this critical point and the second derivative, the adhesion energy for the two films was calculated using different formulae from Table I, assuming a value for Poisson's ratio of 0.3. The results are compiled in Table II. Three of the equations in

TABLE II
Calculated values of the adhesion energy, W [N/m], from blister test experiments employing different formulae from Table I and literature data[16] from blister tests and peel tests. The indicated limits are standard deviations. Values in normal (Roman) type are obtained from our experiments, those in *italic* type are taken from Gent & Lewandowski.[16] Material constants used are indicated below.

Method	Tap #35 (3M)		Tape #375 (3M)	
	On Teflon	On Plexiglas	On Teflon	On Plexiglas
Hinkley				
$W=0.25P_{cr}h_{cr}$	10 ± 2	8 ± 1	12 ± 4	45 ± 9
Takashi & Yamazaki				
$W=\left(\frac{P_{cr}^4a^4}{18.2Et}\right)^{1/3}$	32 ± 12	25 ± 3	30 ± 10	114 ± 10
Gent & Lewandowski				
$W=\left(\frac{P_{cr}^4a^4}{17.4Et}\right)^{1/3}$	33 ± 10	25 ± 3	31 ± 11	116 ± 10
	18.5 ± 2	*26 ± 3*	*22.2 ± 1*	*150 ± 8*
$W=0.65P_{cr}h_{cr}$	25 ± 6	22 ± 3	32 ± 10	116 ± 20
	14.5 ± 5	*24.5 ± 3*	*26.9 ± 1*	*154 ± 8*
Briscoe & Panesar				
$W=\left(\frac{P_{cr}^4a^4}{576(1-\nu)^2Et}\right)^{1/3}$	13 ± 5	10 ± 1	12 ± 5	46 ± 4
Peel test	*46.2 ± 1.5*	*45.2 ± 3*	*95.5 ± 6*	*228 ± 12*
2nd derivative, Eq. (12)	9 ± 5	10 ± 5	15 ± 8	26 ± 15

$E=5.0$ MPa (#35) and 1.2 GPa (#375), $\nu=0.3$

Table I, those of Williams[4] and Andrews and Stevenson[13] were not considered because they are appropriate only for the case of a clamped plate, *i.e.*, where the deflection of the blister's center is small compared with the thickness of the film. That is not the case here.

The values obtained for the adhesion energy by application of the classical formulae to our measurements with both films are in broad agreement with those of Gent and Lewandowski.[16] We deduce from this that the experimental setup and the test samples chosen are sound and suitable for comparing methods of calculation of adhesion parameters. Usually there is a difference in values for the adhesion energy between the classical formulas when all are applied to the same data. Values for the adhesion energy deduced by the second derivative method are within this range. As we discussed above, the fracture energy measured by all mechanical methods consists, to a significant extent, of irreversible plastic work and dissipation. In peel tests, more energy is dissipated than in blister tests and adhesion energies for the same films obtained by a peel test[16] at an angle of 90° are twice as large as values obtained with classical formulae[16] (see Table II).

The most attractive feature of the new method is that we characterize the delamination process with the maximum bending moment (see Table III), *i.e.*, the mechanical

TABLE III
Maximum bending moment, $10^3 \times M_{max}$ [N·m], for the 3M Tapes #35 and #375 on Teflon and Plexiglas, calculated with Eqs. (8) & (9). The indicated limits are standard deviations

		Substrate	
		Teflon	Plexiglas
Tape	#35	2.2 ± 0.7	2.3 ± 0.8
	#375	11.8 ± 3.6	15.6 ± 4.5

cause of delamination, avoiding, in contrast to the energy balance approach, the necessity of taking account of the contribution of the dissipative part. In addition, the new approach presented here is based on measurements close to the joint, again in contrast to all other methods that generally manipulate the energy stored in the deformation of the entire blister up to the delamination.

CONCLUSIONS

A new method proposed here is based on a simple application of the theory of elasticity near the separation line, *i.e.*, near the edge of the blister: the maximum bending moment, M_{max}, is deduced from the curvature of the blister as it bends away from the flat substrate, thus directly yielding values of practical interest and allowing a realistic description of the engineering quality of the joints between materials.

To test our method we carried out blister test experiments with a sensitive profilometer on systems already investigated by Gent and Lewandowski.[16] When we take the classical experimental values from our data, *i.e.*, P_{cr}, the pressure of the injected fluid immediately before delamination of the blister, h_{cr}, the height at the center of the blister immediately before delamination, and a, the radius of the blister, values in broad agreement with Gent and Lewandowski are obtained. Other formulae from the literature, applied to the same data, provide a rather large spread of values for the adhesion energy, however. Values for adhesion energy calculated from the curvature of the film just before delamination agree best with those of Hinkley[14] and Briscoe and Panesar.[17]

It is probably appropriate here to return once more to our use of elastic plate theory in a blister test. Indeed, the blisters as a whole (especially for tapes employed as test samples) behave largely as membranes. Close enough to the separation line, however, they, as any clamped elastic sheet, show plate characteristics. The excellent fit of an elastic plate shape to the experimental deflection of the film near the debonding edge (Fig. 3) is a clear indication of this. Actually, if the blisters behaved like membrances in this region, without flexural rigidity at all, there would have to be a distinct change in the first derivative $\partial z/\partial r$ at the edge, and a concomitant observable quasi-singularity at $r = a$. We conclude that, near the edge, the blisters behave as plates.

Acknowledgements

Support from the Start-Programm für Werkstofforschung of the Swiss Federal Council and of Gurrit-Essex AG, Pfäffikon, Schwyz, is gratefully appreciated. The authors are also happy to acknowledge the valuable assistance of Dr. C. D. Bugg in developing hardware and of N. F. Staeheli for help in developing software.

References and Notes

1. A. J. Kinloch, *Adhesion and Adhesives: Science and Technology* (Chapman and Hall, London, 1987).
2. A. A. Griffith, *Phil. Trans. R. Soc. Lond.* **A221**, 163 (1920).
3. G. C. Sih, *Mechanics of Fracture: Initiation and Propagation. Surface and volume energy density applied as failure criterion* (Kluwer Academic Publishers, Dordrecht, 1991).
4. M. L. Williams, *J. Appl. Polym. Sci.* **13**, 29 (1969).
5. S. Wu, *Polymer Interface and Adhesion* (Marcel Dekker, Inc., New York and Basel, 1982).
6. G. P. Anderson, S. J. Bennet and K. L. DeVries, *Analysis and Testing of Adhesive Bonds* (Academic Press, New York, 1977).
7. K. L. Mittal, *Electrocomponent Science and Technology* **3**, 21 (1976).
8. H. Dannenberg, *J. Appl. Polym. Sci.* **5**, 125 (1961).
9. M. G. Allen and S. D. Senturia, *J. Adhesion* **25**, 303 (1988).
10. D. A. Dillard and Y. Bao, *J. Adhesion* **33**, 253 (1991).
11. M. Fernando and A. J. Kinloch, *Int. J. Adhesion and Adhesives* **10**, 69 (1990).
12. M. G. Allen, M. Mehregany, R. T. Howe and S. D. Senturia, *Appl. Phys. Lett.* **51**, 241 (1987).
13. E. H. Andrews and A. Stevenson, *J. Mater. Sci.* **13**, 1680 (1978).
14. J. A. Hinkley, *J. Adhesion* **16**, 115 (1983).
15. M. Takashi and K. Yamazaki, *Proc. Jpn. Congr. Mater. Sci.*, 960 (1987).
16. A. N. Gent and L. H. Lewandowski, *J. Appl. Polym. Sci.* **33**, 1567 (1987).
17. B. J. Briscoe and S. S. Panesar, *Proc. R. Soc. Lond.* **A433**, 23 (1991).
18. J. W. Obreimoff, *Proc. Royal Soc. (London)* **A127**, 290 (1930).
19. L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon, New York, 1986), p. 38–45.
20. S. Timoshenko, *Strength of Materials, Part II* (Macmillian and Co, London, 1930).
21. C. D. Bugg and P. J. King, *J. Phys. E: Sci. Instrum.* **21**, 147 (1988).
22. The precision of centering the sample with its hole under the scan line is not crucial. A blister with the shape of Eq. (7) near its boundary ($r = a$) and the center of which is located at a distance y_0 from the scan line in the y-direction (see Fig. 2) yields an apparent second derivative on scanning of

$$\left.\frac{\partial^2 z}{\partial x^2}\right|_{r=a} = \left.\frac{\partial^2 z}{\partial r^2}\right|_{r=a}\left(1 - \frac{y_0^2}{a^2}\right)$$

For a blister of radius 3 mm, for instance, an improbably large misalignment of 1 mm would lead to a second derivative and, hence, the values for M_{max} and W, too small by 11%; such an error is comparatively insignificant.